Little String Theory

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Outline

- 6d super conformal field theories
- 6d little string theories
    - T-duality in type IIA and type IIB LST
    - (1,0) LSTs & T-duality
- Conclusion

Higher Dimensional Theories

- Include gravity
  - 10-dim superstring theories with g_s, l_s: type I, type IIA, IIB, HO, HE
  - 11-dim M-theory with l_p: M2 and M5 branes, strongly interacting
    - low energy dynamics = 11d supergravity
- without gravity
  - 5,6-dim superconformal field theories: non-Lagrangian theory, strongly interacting
  - 6-dim little string theories with l_p: strongly interacting

6d SCFT

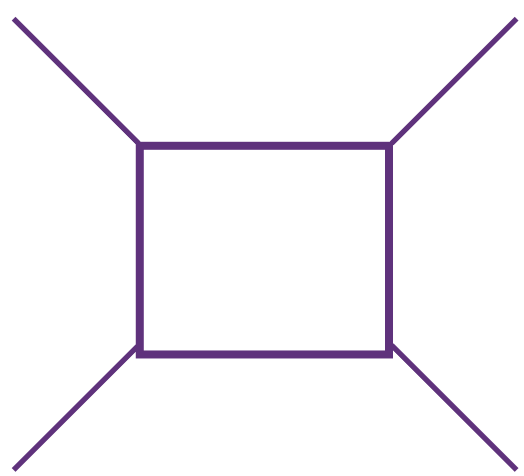
Nahm’78,Witten’95,Strominger’95,Ganor&Hanany’96….

- N=(2,0) 16 supersymmetric theories
- A_n,D_n,$E_{7,8,9}$ types: type IIB on ADE-type ALE space: $\mathbb{C}^2/\Gamma_G$
- A_n or SU(n+1) type on n+1 M5 branes
- B,Φ_I(I=1,2,3,4,5), Ψ: selfdual tensor H=dB=*H, 3+5=8
- *d*H=J: self-dual strings=M2 branes
- tensionless strings, local field theories

6d (1,0) SCFTs

Seiberg'96,Danielsson et.al.'97

- It is made of vector, hyper, and tensor multiplets
- The gauge anomaly due to vector and hypermultiplet fermion 1-loop should vanish.
- The gauge anomaly polynomial is made of two pieces. The first one should vanish. The second one can be removed with coupling to tensor multiplet and using the Green-Schwartz mechanism



	spinor helicity
vector	(0,1)
hyper	(1,0)
tensor	(1,0)
supercharge	(1,0)

6d (1,0) SCFTs

Seiberg'96,Danielsson et.al.'97

$$\mathrm{Tr}_R F^4 = \alpha_R \mathrm{tr} F^4 + c_R (\mathrm{tr} F^2)^2$$

$$\alpha_R = 0 \ for \ SU(2), SU(3), G_2, SO(8), F_4, E_6, E_7, E_8$$

$$c_{\mathrm{tot}} = \left[c_{Ad} - \sum_{R \ \mathrm{matter}} c_R \right] \geq 0$$

- The gauge anomaly polynomial is made of two pieces. α_R should vanish when all contributions are included. c_R >0 can be removed with coupling to tensor multiplet and using the Green-Schwarz mechanism.

$$H^2 + \sqrt{c_R}(B \wedge \mathrm{tr} F \wedge F + \Phi F^2)$$

- If $\alpha_{tot=0}$, c_{tot}=0, one can have a (1,0) SYM with the inverse coupling $1/l_s^2$: instanton strings = **little string theory**

6d (1,0) SYM with a_R=0 + Green-Schwarz+ global anomaly

- SU(2): n_2=4,10,16 (LST)
- SU(3): n_3=0,6,12,18 (LST)
- G_2 : n_7=1,4,7,10 (LST)
- SO(8): n(v,c,s) =0,1,2,3,4(LST)
- F_4: n_{26}=0,1,2,3,4,5(LST)
- E_6: n_{27}=0,1,2,3,4,5,6(LST)
- E_7: $n_{56/2}$=0,1,2,3,4,5,6,7,8(LST)
- E_8

N M5s on ADE singularities

N array of (G,G) conformal matters with G in insides get gauged

A_N = N D6 with n NS5 : [SU(N)]-SU(N)-SU(N)…-[SU(N)]

D_{N+4}= [SO(2N+8)]-Sp(N)-SO(2N+8)-Sp(N)….Sp(N)-[SO(2N+8)]

E_6: +.. (N-1) E_6 gauge theories +2 E_6 global symmetries:

[E_6] -T-SU(3)-T-E_6-T -SU(3)-… … -SU(3)-T- [E_6]

E_7: (N-1) E7 gauge theories+ 2E_7 global symmetries..,

[E7]-T-SU(2)-SO(7)-SU(2)-T- E7-T-……… -T-[E7]

E_8: (N-1) E_8 gauge theories +2E_8 global symmetries

[E_8]-T-T-SU(2)-G_2-T-F_4-T-G_2-SU(2)-T-T-E8-T-……-T-T-[E8]

6d maximally susy LST

Aharony-Berkooz

- The low energy dynamics of NS5 branes + fundamental strings of type IIA & type IIB in the limit where gravity decouples: fix l_s and take $g_s=0$
- There is only one scale, the little string tension $1/l_s^2$.
- type IIA LST with (2,0) supersymmetry: on N NS5 branes + F strings in type IIA or F strings in type IIB A_{N-1} (DE) singularity
- type IIB LST with (1,1) supersymmetry: on N NS5 branes + F strings in type IIB or F-strings in type IIA A_{N-1} (DE) singularity
- Low energy dynamics of type IIA LST is the 6d (2,0) SCFT of ADE type symmetry.
- Low energy dynamics of type IIB LST is the 6d (1,1) Yang-Mills theory of any compact group with coupling $g_{YM}^2=l_s^2$

Little String Theories

- string theory with a scale
- no energy momentum tensor or local operators (T-duality)
- (2,0), (1,1), (1,0) supersymmetries
- low energy dynamics = free or superconformal field theory
- NS 5 branes in type IIA,IIB,HO,HE string theories
- type IIA,IIB,I on ADE singularity C^2/Γ_G

6d (1,0) LST

Seiberg97

Bhardwaj et.al, 1511.05565

- vectors+ hypers+ tensors
- a_R vanishes for each gauge group.
- c_R vanishes one less number of groups
- on NS5 branes exploring ADE ALE space in type IIA or type IIB
- on NS5 branes in E_8xE_8 or SO(32) Heterotic string theories

T-duality between IIA and IIB LST

J.Kim,S.Kim,KL'15

- type IIA LST on compactified on a circle S with momentum p and winding w are T-dual to type IIB LST compactified on the dual circle S with momentum w and winding p.

- LST on the world volume R^{1+5} of NS5 branes, Partition function along R^{1+1} with momentum and strings, Localization on R^4 on the world volume and R^4 of transverse R5 with turning on chemical potentials: $\varepsilon_1,\varepsilon_2,\varepsilon_3,\varepsilon_4$, For a given super-charge, there is only three independent ones, say their sum vanishes.

- elliptic genus of instanton strings and M-strings are needed to show this.

$$Z_{\text{inst}}^{\text{IIB}}(\alpha_i, \epsilon_\pm, m; q, w) = \text{Tr}\Big[(-1)^F w^k q^{H_L} \bar{q}^{H_R} e^{2\pi i \alpha_i \Pi_i} e^{2\pi i \epsilon_-(2J_{1L})} e^{2\pi i m(2J_{2L})} e^{2\pi i \epsilon_+(2J_{1R}+2J_{2R})}\Big]$$

- type IIB LST on a circle is characterized by the gauge holonomy $(\alpha_1,\alpha_2,\ldots,\alpha_N)$, leading to the fractionalization of momentum: instanton strings in 6d (1,1) SU(N) strings

- type IIA LST has fractionalized strings as NS5 branes on `M-circle' can be connected by M2 branes. : M-strings Haghighat et.al. 1305,1310

- (0,4) gauged linear sigma model, elliptic genus: Benini,Eager,Hori,Tachikawa 1305,1308

type IIB LST

- 6d (1,1) Yang-Mills theory has instanton strings. They are little strings of the theory.

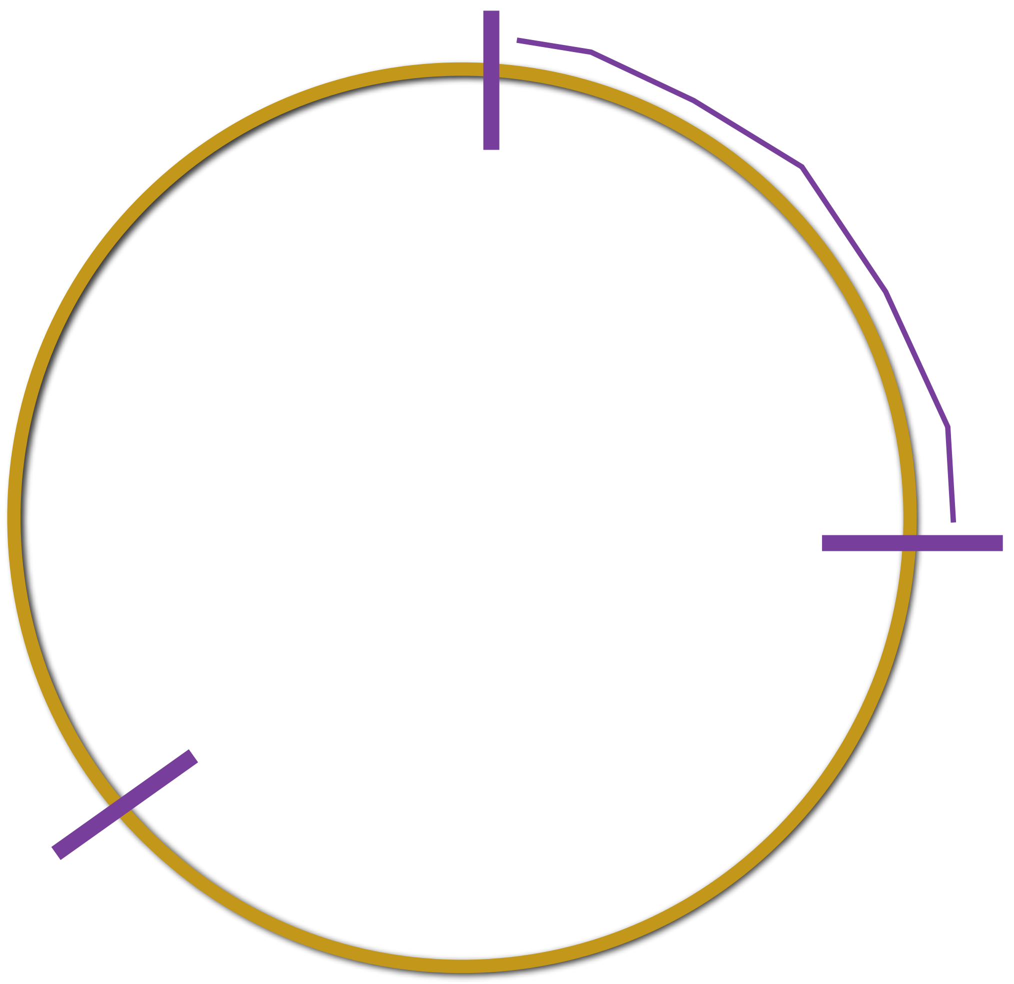
- Instanton string dynamics can be characterized by (4,4) ADHM string model. Its elliptic genus can be obtained by simply generalizing the 5d result to elliptic case. k-instanton string contribution is characterized by Young diagrams

- There is also perturbative contribution from 6d YM theory which counts the massless modes along the string direction.

- The total contribution is a product of perturbative and stringy contributions.

$$Z_{\rm IIB}(\alpha_i,\epsilon_\pm;q,w)=Z_{\rm KK}^{\rm IIB}(\epsilon_\pm,m;q)Z_{\rm string}^{IIB}(\alpha_i,\epsilon_\pm,m;q,w)$$

$$Z_{\rm string}^{IIB}(\alpha_i,\epsilon_\pm,m;q,w)=\sum_{n=0}^{\infty}w^n Z_n(\alpha_i,\epsilon_\pm,m;q)$$



type IIA LST

Aharony-Berkooz'99,J.Kim,S.Kim,KL'15

- NS5 branes on M-circle= M5 branes at position $(\alpha_1,\alpha_2,\ldots,\alpha_N)$ on M-circle

- M2 branes connecting these M5 branes leads to fractionally winded strings. Elliptic genus for M-strings.

Haghighat,Iqbal,Kozcaz,Lockhart,Vafa (2013)

- There is also perturbative contribution from 6d (2,0) theory in the Coulomb branch.

- The total contribution is a product of perturbative and stringy contributions.

$$Z_{\rm IIA}(\alpha_i,\epsilon_\pm;q',w') = Z^{\rm IIA}_{\rm KK}(\epsilon_\pm,m;q')Z^{IIA}_{\rm frac\ string}(\alpha_i,\epsilon_\pm,m;q',w')$$

$$Z^{IIA}_{\rm frac\ string}(\alpha_i,\epsilon_\pm,m;q,w) = \sum_{n_I=0}^{\infty}\prod_{I=1}^{n} e^{n_I(\alpha_I-\alpha_{I+1})+\cdots+n_N(\alpha_N-\alpha_1+\ln w')}Z_n(\alpha_i,\epsilon_\pm,m;q)$$

Comparison

J.Kim,S.Kim,KL'15

- One has to take care of the mode where strings get bounded and unhinged from NS5 branes.

$$\hat{Z}_{\text{IIA}}(\alpha_i, \epsilon_{1,2}, m, q', w') = \frac{Z_{\text{IIA}}(\alpha_i, \epsilon_{1,2}, m, q', w')}{Z_{\text{extra}}(w')}.$$

- We have checked the T-duality up to three NS5 branes

- For the single NS5 brane, the contributions of type IIA and type IIB are identical, implying the symmetry under the exchange of p' and w'

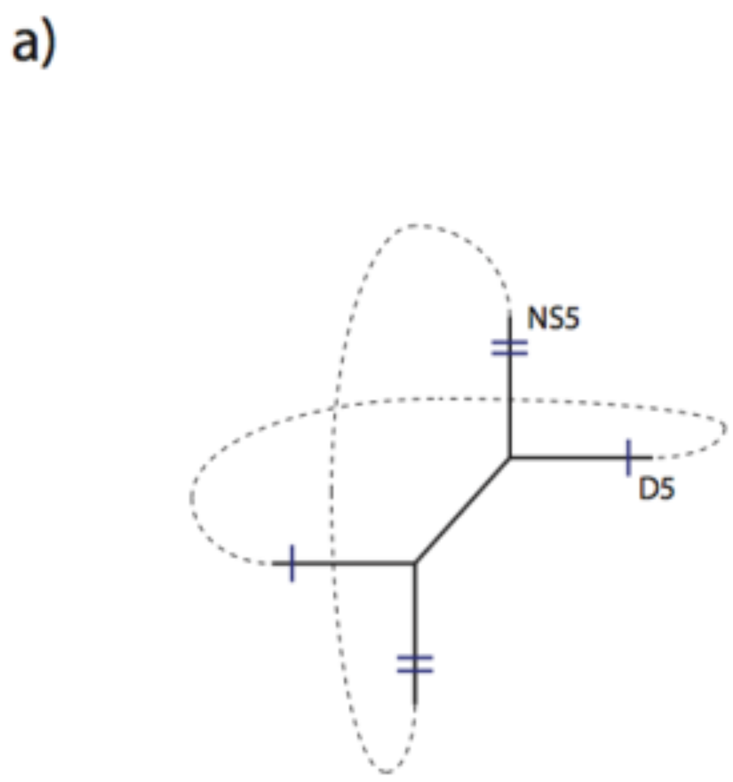
$$Z_{\text{IIA}}(\epsilon_\pm, m; q', w') = PE\Big[I_-(\epsilon_\pm, m) z_{\text{sp}}(\epsilon_\pm, m, q', w')\Big]$$

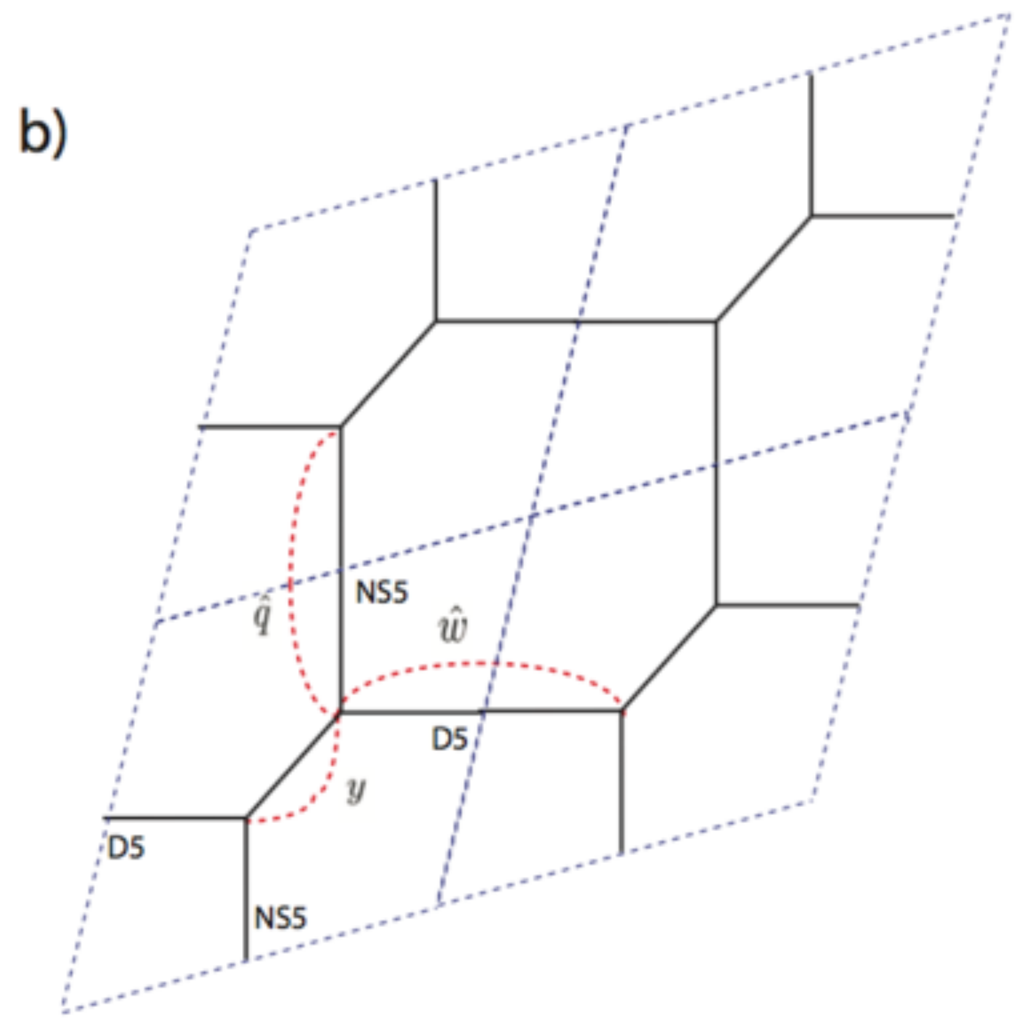
symmetric under q’, w’ exchange

$$z_{\rm sp}(\epsilon_\pm, m; q', w') = (q'+w') + (q'^2+w'^2) + (q'w')\left[tu + \frac{t}{u} + \frac{1}{tu} + \frac{u}{t} - uy - \frac{y}{u} - \frac{u}{y} - \frac{1}{uy}\right]$$
$$+ q'^3 + w'^3 + (q'^2w' + q'w'^2)\Big[t^2u^2 + \frac{t^2}{u^2} + \frac{u^2}{t^2} + \frac{1}{t^2u^2} + t^2 + \frac{1}{t^2} - tu^2y - \frac{ty}{u^2} - \frac{tu^2}{y} - \frac{t}{u^2y}$$
$$- \frac{y}{tu^2} - \frac{u^2}{ty} - \frac{1}{tu^2y} - \frac{u^2y}{t} + tu + \frac{t}{u} + \frac{1}{tu} + \frac{u}{t} - 2ty - \frac{2t}{y} - \frac{2}{ty} - \frac{2y}{t} + 2u^2 + \frac{2}{u^2} - uy - \frac{y}{u}$$
$$-\frac{u}{y} - \frac{1}{uy} + y^2 + \frac{1}{y^2} + 4\Big] + (q'^4 + w'^4) + (q'^3w' + q'w'^3)\Big[t^3u^3 + \frac{t^3}{u^3} + \frac{u^3}{t^3} + \frac{1}{t^3u^3} + t^3u + \frac{t^3}{u}$$
$$+ \frac{u}{t^3} + \frac{1}{t^3u} - t^2u^3y - \frac{t^2y}{u^3} - \frac{t^2u^3}{y} - \frac{t^2}{u^3y} - \frac{u^3y}{t^2} - \frac{y}{t^2u^3} - \frac{u^3}{t^2y} - \frac{1}{t^2u^3y} + t^2u^2 + \frac{t^2}{u^2} + \frac{u^2}{t^2}$$
$$+ \frac{1}{t^2u^2} - 2t^2uy - \frac{2t^2y}{u} - \frac{2t^2u}{y} - \frac{2t^2}{uy} - \frac{2uy}{t^2} - \frac{2y}{t^2u} - \frac{2u}{t^2y} - \frac{2}{t^2uy} + 2t^2 + \frac{2}{t^2} + 2tu^3 + \frac{2t}{u^3}$$

$$t = e^{2\pi i\epsilon_+},\ u = e^{2\pi i\epsilon_-},\ y = e^{2\pi i m}.$$

- There is also triality for the single NS5 brane case. There is a p-q 5 brane web for mass-deformed case





$$\hat{q} = qy^{-1}\ , \quad \hat{w} = wy^{-1}\ .$$

triality under exchange of $y,\ \hat{q},\ \hat{w}$

Hollywood,Iqbal,Vafa(2008)

$$\tilde{Z}(\epsilon_\pm;\hat{q},\hat{w},y) = PE\Big[I_{\rm com}\tilde{z}_{\rm sp}(\epsilon_\pm;\hat{q},\hat{w},y)\Big],$$

$$\begin{aligned}
\hat{z}_{sp}(\epsilon_\pm;\hat{q},\hat{w},y) &= \hat{q}+\hat{w}+y-(u+u^{-1})(\hat{q}\hat{w}+\hat{q}y+\hat{w}y)+\frac{(1+u^2)(t+u+t^2u+tu^2)}{tu^2}\hat{q}\hat{w}y \\
&+(\hat{q}^2\hat{w}+\hat{q}\hat{w}^2+\hat{q}^2y+\hat{q}y^2+\hat{w}^2y+\hat{w}y^2)-(u+u^{-1})(\hat{q}^2\hat{w}^2+\hat{q}^2y^2+\hat{w}^2y^2) \\
&-\frac{(u^2+1)\left(t^2\left(u^2+1\right)+2tu+u^2+1\right)}{tu^2}\hat{q}\hat{w}y(\hat{q}+\hat{w}+y) \\
&+(\hat{q}^3\hat{w}^2+\hat{q}^2\hat{w}^3+\hat{q}^3y^2+\hat{q}^2y^3+\hat{w}^3y^2+\hat{w}^2y^3)+\frac{(1+u^2)(t+u+t^2u+tu^2)}{tu^2}\hat{q}\hat{w}y(\hat{q}^2+\hat{w}^2+y^2) \\
&+\frac{t^4\left(u^5+u^3+u\right)+t^3\left(u^6+4u^4+4u^2+1\right)}{t^2u^3}\hat{q}\hat{w}y(\hat{q}\hat{w}+\hat{q}y+\hat{w}y) \\
&+\frac{t^2\left(3u^4+7u^2+3\right)u+t\left(u^6+4u^4+4u^2+1\right)+u^5+u^3+u}{t^2u^3}\hat{q}\hat{w}y(\hat{q}\hat{w}+\hat{q}y+\hat{w}y) \\
&-(u+u^{-1})(\hat{q}^3\hat{w}^3+\hat{q}^3y^3+\hat{w}^3y^3)
\end{aligned}$$

T-duality in HO & HE LST

- HO (SO(32)) LST on compactified on a circle S with momentum p and winding w are T-dual to HE(E_8xE_8) LST compactified on the dual circle S with momentum w and winding p.

- 6d gauge theory of LST on N NS5 branes in HO(SO(32)) = 6d (1,0) Sp(N) SYM with 16 fundamental hypermultiplet + 1 anti-symmetric hyper

    - torus partition function with instanton strings: fractional linear momentum due to the gauge holonomy

- 6d theory of LST on N NS5 branes in HE(E_8xE_8) is E8 SCFTs.

    - HE LST has fractionalized fundamental strings as M2 branes connects M9 walls to M5 branes and M5 branes… and then M9 brane.

HO LST

- on D1 on type I (SO(32)) in 2d (0,4) language = vector in O(k) adj + fermi in O(k) adj+ hyper in O(k) symmetric + twisted hyper in O(k) symmetric + fermi in O(k)xSO(32) bifundamental

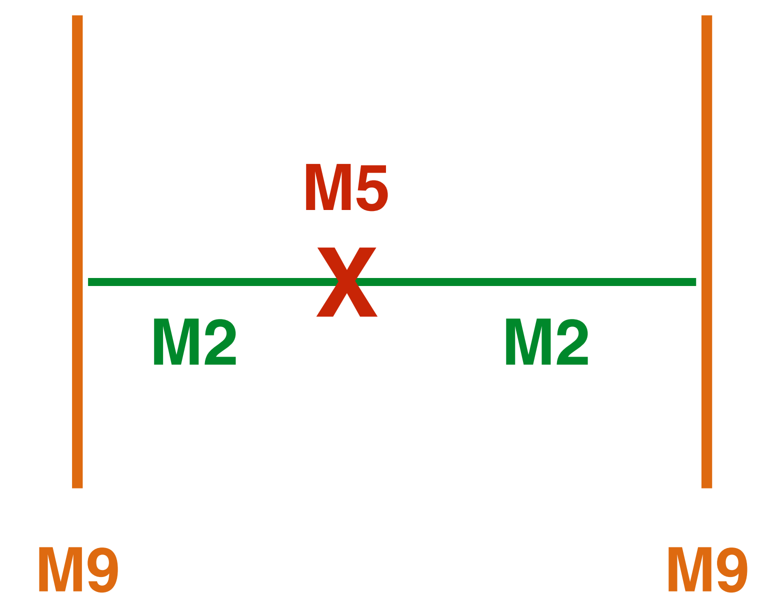
- with additional N D5 branes with : hyper in O(k)x Sp(N) bifundamental + torus partition function with instanton strings: fractional linear momentum due to the gauge holonomy

- Elliptic genus of the fundamental strings for HO LST and HO:

$$I_1 = \frac{-\eta^4}{\theta_1(\epsilon_1)\theta_1(\epsilon_2)\theta_1(-\epsilon_+ \pm \mathfrak{m})} \sum_{I=1}^{4} \frac{\theta_I(\mathfrak{m} \pm a)}{\theta_I(\epsilon_+ \pm a)} \prod_{l=1}^{16} \frac{\theta_I(m_l)}{\eta}$$

$$I_1' = \frac{-\eta^4}{\theta_1(\epsilon_1)\theta_1(\epsilon_2)\theta_1(-\epsilon_+ \pm \mathfrak{m})} \prod_{l=1}^{16} \frac{\theta_I(m_l)}{\eta}$$

HE LST



- O8+8D8 + k_1 D2 + NS5+ k_2 D2+ O8+D8

- SO(8) - O(k_1)- bifundamental-O(k_2)-SO(8)

- 6d theory of LST on N NS5 branes in HE(E_8xE_8) is E8 SCFTs.

  - HE LST has fractionalized fundamental strings as M2 branes connects M9 walls to M5 branes and M5 branes… and then M9 brane.

- E-strings elliptic genus: Kim,Kim,KML,Park,Vafa1411, Kim,Kim,Lee1510

- Single HE LST, HE

$$+\frac{\eta^4}{\theta_1(\epsilon_1)^2\theta_1(-\epsilon_2)^2}\frac{\theta_1(-\mathrm{m}\pm\epsilon_-)}{\theta_1(-\mathrm{m}\pm\epsilon_+)}\frac{A_1\times A_1}{\eta^{16}}$$

$$\frac{-\eta^4\cdot\Theta^{E_8\times E_8}}{\theta_1(\epsilon_{1,2})\theta_1(-\epsilon_+\pm\mathrm{m})}$$

Conclusion

- There are a lot more to be learned about 6d SCFTs and LST

- UV of 6d (1,1) SYM is completed in LST.

- Elliptic genus for selfdual strings is a powerful tool. But it is not easy to write down the UV theory for some cases.

- There are more approaches to 6d theories like bootstrap, gravity dual, DLCQ, effective action.

- There is also a powerful relation to lower dimensional physics (5,4,3,2,1,0).